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STRUCTURI DIRAC PE VARIETĂŢI FINIT ŞI INFINIT DIMENSIONALE

Summary

Conducător ştiinţific:
Prof. univ dr. MIHAI ANASTAIEI
Universitatea "Alexandru Ioan Cuza", Iaşi

Doctorand:
VLAD-AUGUSTIN VULCU
Universitatea "Alexandru Ioan Cuza", Iaşi

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The study of a mechanical physical phenomenon by applying the classical theory of mechanical systems is done by considering either the Lagrangian formalism, either in terms of the Hamiltonian formalism. The first of these is based on the use of different variational principles, and therefore it can be easily generalized to the relativistic case, while the second one is based on the concept of energy and it is often used in quantum mechanics.

If we were to denote by $M \subseteq \mathbb{R}^m$ the domain of the physical states corresponding to the physical phenomenon that is intended to be studied, then according to the Lagrangian formalism there exists defined a regular function $L : TM \to \mathbb{R}$, $L = L(x, y)$, of class C^2, on M, where TM denotes the set of positions (x) and of directions (y) corresponding to the physical phenomenon. From a historical point of view, among the first Lagrangians considered there is the difference between the kinetic energy E_c and the potential energy E_p of a physical phenomenon. By duality, for each Lagrangian $L = L(x, y)$, there exists a Hamiltonian $H : T^{\star}M \to \mathbb{R}$, given by $H(x, p) = y^i p_i - L(x, y)$ where $T^{\star}M$ denotes the set of positions (x) and momenta (p) corresponding to a physical phenomenon and where $y = (y^i)$ is the solution of the nonlinear equation $p_i = \frac{\partial L}{\partial y^i}(x, y)$.

These formalisms have led to the introduction of two remarkable geometries, the geometry of Lagrange spaces and the geometry of Hamilton spaces. On one hand, the Lagrange geometry is a pair (M, L) where M is a differentiable manifold of finite dimension (or modeled by a Banach space) and $L : TM \to \mathbb{R}$ is a regular map of class C^2, on M. The set TM denotes the tangent bundle of M. On the other hand a Hamilton space is a pair (M, H) where M a manfold and $H : T^{\star}M \to \mathbb{R}$ is a regular map of class C^2, on M. The set $T^{\star}M$ denotes the cotangent cotangent bundle of M. These spaces and their properties, are widely studied in articles and monographs as [**24-27**].

In the Hamiltonian formalism, in addition to using of the Hamilton function H in the study of a physical phenomenon, there was considered the natural symplectic structure of $T^{\star}M$, which made possible a new approach to the study of different mechanical systems, hence a mechanical system is a triplet (M, H, ω) where M is a manifold, H is a real valued smooth function defined on M and ω is a symplectic 2-form on M, i.e. ω is closed and non-degenerate. Thus, the evolution curves of a mechanical system (M, H, ω) are given by the trajectories of a vector field $X \in \chi(M)$, which is the only solution of the equation $i_X\omega = -dH$.

Afterwards, it was showed that these two geometries could not be used to study non-holonomic and mechanical systems with constraints, This led to the introduction of pre-symplectic structures, on one hand, and on the other, to Poisson structures.

A pre-symplectic structure is a pair (M, ω) where M is a manifold and ω is a degenerate and closed 2-form on M. The reason that led to the introduction of these structures is as follows. Let (M, ω) a symplectic manifold, of dimension $2m$, and let $N \subset M$ be a submanifold determined by conditions $\varphi_1(x) = ... = \varphi_m(x) = 0$. Then the restriction of ω, to N, is still closed, but $\omega|_N$ could be degenerate. *P.A.M Dirac* had proved that ω is non-degenerate if and only if the matrix $\Phi = \left\| \left\{ \varphi_i, \varphi_j \right\} \right\|$ is non-degenerate, and, furthermore, if $\{,\}_M$ is a Poisson bracket on M, then the pairing $\{,\}_N$, given by$\{f, g\}_N$

$= \{f,g\}_M - \sum_{j,k}\{f,\varphi_j\}\, c_{jk}\,\{\varphi_k, g\}$, where $C = \|\{c\_\{jk\}\}\|$ is the inverse of Φ, is a Poisson bracket on N, hence the pair $(C^\infty(N), \{,\}_N)$ is a Poisson algebra. Although $\omega|_N$ is degenerate, one can still construct a Poisson algebra.

A Poisson structure is a pair $(M, \{,\})$, such that if we denote by $C^\infty(M)$ the set of smooth real functions on M, then the pair $(C^\infty(M), \{,\})$ is a Poisson algebra, i.e. the map $\{,\}$ is bilinear, skew-symmetric, satisfies the Jacobi identity and Leibniz's property. These structures were introduced by *A. Lichnerowicz* in [19], and subsequently studied by *A. Weinstein* in [46], *A.A. Kirillov* in [16], *V. Guillemin* and *S. Sternberg* in [13] or by *I. Vaisman* in [40].

In 1967, in [30], *J. Pradines* introduces the notion of Lie algebroid as the infinitesimal version of a Lie groupoid. Therefore a Lie algebroid over a Banach manifold M is a triple $(E, [,]_E, \rho_E)$ where the pair (E, ρ_E) is an anchored Banach bundle, $(E, [,]_E)$ is a Lie algebra and the anchor ρ_E induces a Lie algebra morphism $\rho_E : \Gamma(E) \to \chi(M)$.

In 1976, in [31], L. Maxim-Răileanu showed that for any Lie algebroid $(E, [,]_E, \rho_E)$ there can be constructuded its graded differential algebra of multilinear and alternate maps, which is denoted by $\Lambda^\star(E) = \Lambda^0(E) \oplus \Lambda^1(E) \oplus ...$ and that the pair $(\Lambda^\star(E), d_E)$ is a differential complex with respect to the Lie algebra of E.

In 1987, *K. Mackenzie*, in [21], gives a first systematic approach to the study of both Lie groupoids and Lie algebroids. As it was only later observed, the notion of Lie algebroid represents a much more general framework than the tangent bundle of a manifold, hence the Lagrangian formalism was naturally extended to the case of the Lie algebroids (*E. Martinez* [23], *M. Anastasiei* [1]). On this occasion there could be studied some of the singular geometric structures (*R.L. Fernandez*, [12]).

In 1988, *T.J. Courant* and *A. Weinstein*, in [9], and later *T.J. Courant,* in 1990, in [8], define the notion of Dirac structure on a finite dimensional vector space V, as a maximal and isotropic linear subspace $D \subset V \times V^\star$ with respect to a bilinear, symmetric and non-degenerate mapping $\langle,\rangle_+$, where $V^\star$ denotes the algebraic dual of V. Then, the two extend this definition to the case of a finite dimensional manifold M, where a Dirac structure is a maximal and isotropic sub-bundle $D \subset TM \oplus T^\star M$ with respect to a bilinear, symmetric and non-degenerate mapping $\langle,\rangle_+$, defined on the sections of $TM \oplus T^\star M$. By considering the graphs of certain bundle morphisms, *T.J. Courant* and *A. Weinstein* show that these structures extend both Poisson and pre-symplectic structures.

Also, in [8], *T.J. Courant* introduces the notion of integrable Dirac structure with respect to a bilinear and skew-symmetric pairing $[,]_C$, which, in general, does not satisfy the Jacobi idenity. In that same article, *Courant* determines necessary and sufficient conditions for a Dirac structure to be integrable,such as, a Dirac structure is integrable if and only if it can be endowed with a Lie algebroid structure. Later it was showed that these structures have many important applications in mechanics and partial differential equations (*I. Dorfmann*, [11], *A. J. van der Schaft*, [34], *van der Schaft* şi *G. Blankenstein*, [35], *Blankenstein* şi *T. S. Raţiu* [6], and others).

In [4], *M. Anastasiei* and *A. Sandovici* have extended the notion of Dirac structure to the case of smooth Banach manifolds and have showed that a Dirac structure is integrable if and only if it can be endowed with a Lie algebroid structure.

Another extenion of the notion of Dirac structure was given by *A. Weinstein*, *P. Xu* and *Z. J. Liu*, in [20] where they introduce the Courant algebroid by means of a definition with five conditions. Later, this definition was used by *M. Gualtieri*, in [14] in order to study the generalized complex structures, and respectively by *D. Roytenberg* in [32], who determines the basic properties these structures and shows the equivalence between the definition given by *A. Weinstein*, *P. Xu* and *Z.J. Liu* and the one proposed by *A. Weinstein* and *P. severa*. Later, *K. Uchino* shows in [37] that in order to define a Courant algebroid only three of the five axioms are requiered. Thus by taking as starting point the definition with three axioms proposed by *F. Keller* and *S. Waldmann*, in [45], together with *M. Anastasiei*, in [5], we have defined the notion of Courant algebroid. In that same paper it was showed that if $(E, [,]_E, \rho_E)$ were a Lie algebroid then the anchored Banach bundle $E \oplus E^\star$ is a Courant Banach vector bundle structure.

In 2002, in [33], *van der Schaft* and *BM Maschke*, by using Stokes's formula and the Poincaré duality principle introduce a new class of Dirac structures, called Stokes-Dirac structures. After that, they show that the equations of electromagnetism, the equations of the ideal telegraphy or vibrating string equation can be derived by considering such structures. A generalization of these structures was later given by *G. Nishida* and *M. Yamakita* in [28], where, by using the Hodge $\star$ operator, the two introduce the extended Stokes-Dirac structures of the type $\star(d\star)^m$ and $d(\star d)^m$ respectively, and for $m = 1$ they derive the Euler-Bernoulli beam equation.

In the following we describe the contents of this thesis, emphasizing on the important results that were obtained.

The first chapter, entitled *Banach Manifolds*, is divided into two sections, *Differential Calculus on Banach Spaces* and *Banach manifolds*.

In the first section we define the notions of Fréchet derivative, higher order Fréchet derivative, directional derivative, partial derivative, C^r map and C^r-diffeomorphism, where $r \geq 1$. Then, the state, without any proof, the following results: The Inverse Mapping Theorem, The Implicit Function Theorem, The Local Surjectivity Theorem and The Local Injectivity Theorem.

In the second section we define the notion of Banach manifold and construct the tangent and cotangent bundles of a Banach manifold. Then, we define the concepts of immersion, submersion and embedding, vector field, 1-form and that of k-form. Afterwards we define the Lie derivative the exterior differential, the inner product and exterior product respectively, and state without any proof the basic properties of these maps.

The second chapter, entitled *Lie Algebroids*, is divided into four sections, *Anchored Vector Bundles*, *Lie Algebroids*, *Differential Calculus on Lie Algebroids* and respectively *Semisprays on Vector Bundles*.

In the first section, we define the notion of anchored Banach vector bundle with respect to a Banach manifold M, as a pair (E, ρ_E) where $\blacksquare : E \to M$ is a Banach vector bundle and $\blacksquare_E : E \to TM$ is a Banach vector bundle morphism, which is also known as an anchor.

In the second section we define the notion of Lie algebroid, as a triple $(E, [,]_E, \rho_E)$, where (E, ρ_E) is an anchored vector bundle, $(E, [,]_E)$ is a Lie algebra and the anchor $\blacksquare_E$ induces a homomorphism of Lie algebras, denoted by $\blacksquare_E : \blacksquare(E) \to \blacksquare(TM)$, such that $[,]_E$ has the Leibniz property with respect to it.

In the third section we define the basic operators of differential calculus on Lie algebroids, i.e. the exterior differential operator d_E, the inner product i_s and Lie derivative L_s, where $s \in \Gamma(E)$. Since $d_E^2 = 0$ one can easily define the cohomology groups of a Lie algebroid. Then we show that for any Lie algebroid $(E, [,]_E, \rho_E)$ its exterior graded algebra $(\blacksquare^\star(E), d_E)$ is a differential complex over the Lie algebra of E and that the Lie derivative commutes with the exterior differential and inner product, i. e. $d_E \circ L_s = L_s \circ d_E$, $i_s \circ L_s = L_s \circ i_s$ and respectively $L_{[s,r]_E} = L_s \circ L_r - L_r \circ L_s$ and $i_{[s,v]_E} = L_s \circ i_r - i_r \circ L_s$.

In the fourth section, we define the notion of semispray with respect to an anchored vector bundle, thus extending the definition given in the case of the tangent bundle and determine the transformation laws obeyed the local coefficients of such an object.

The third chapter, entitled *Dirac Structures*, is divided into three sections, *Dirac Structures on Vector Spaces*, *Dirac Structures on Lie Algebroids* and respectively *Dirac Structures on Courant Algebroids*.

In the first section we define the notion of Dirac structure on a vector space V as a maximal and isotropic liniar subspace D of $V \times V^\star$, with respect to a bilinear, symmetric and non-degenerate mapping $\langle , \rangle_+$, defined on $V \times V^\star$, where $V^\star$ denotes the algebraic dual of V, and show that the graphs of the skew-symmetric maps $A : V \to V^\star$ and $B : V^\star \to V$ are Dirac structures, on V. In the last part of this section we have the characteristic equations of a Dirac structure.

In the second section we define the notion of Dirac structure with respect to a Lie algebroid. Let $(E, [,]_E, \rho_E)$ be a Lie algebroid and let $E^\star$ denote its dual . We consider the anchored bundle $E \oplus E^\star$, which we endow with a bilinear, symmetric and non-degenerate pairing $\langle , \rangle_+$, given by $\langle (s, \alpha), (v, \beta) \rangle_+ = \frac{1}{2}(i_s\beta + i_v\beta\alpha)$, where (s, α), $(v, \beta) \in E \oplus E^\star$. Let $D^\perp$ denote the ortho-complement of a vector sub-bundle $D \subset E \oplus E^\star$, with respect to $\langle , \rangle_+$. Hence, D a Dirac structure on the Lie algebroid E if $D = D^\perp$. If we assume that E is reflexive then I showed that the graphs of the skew-symmetric morphisms $A : E \to E^\star$and $B : E^\star \to E$, respectively, are Dirac structures on E. An important example of a skew-symmetric morphism is $s \in E \to i_s\blacksquare \in E^\star$, where ω is a 2-form on E. In this case, it is no longer necessary that E be reflexive, thus the sub-bundle $D_\omega = \{(s, i_s\blacksquare) | s \in E\}$ is a Dirac structure on E. Here , we give some examples of Dirac structures derived by using non-linear connections on both the tangent bundle TM, and on the cotangent bundle $T^\star M$ of a manifold M. Next, i determine the characteristic equations of a Dirac structure on a Lie algebroid and write these equations for some of the structures mentioned above.

The definition of an integrable Dirac structure on a Lie algebroid is given by extending the Courant bracket $[,]_C$, introduced by *Courant* in [8]. In this case, the Courant bracket $[,]_C$ is given by $[(s, \alpha), (v, \beta)]_C = ([s, v]_E, L_s\beta - L_v\alpha + \frac{1}{2}d_E(i_v\alpha - i_s\beta))$, where (s, α), (v, β)

$\in \Gamma(E \oplus E^\star)$. By the properties of exterior differential, inner product and Lie derivative it follows that $[,]_C$ is linear skew-symmetric but, in general, it does not satisfy the Jacobi identity. Thus it is said that a Dirac structure $D \subset \left(E \oplus E^\star, \langle , \rangle_+\right)$ is integrable if it is closed with respect to the Courant bracket.

In the following, I determined several conditions equivalent to integrability of Dirac structures. I denoted by $J = (J_1, J_2) : \Gamma(E \times E^\star)^3 \rightarrow \Gamma(E \oplus E^\star)$ the jacobiator of the Courant bracket Courant $[,]_C$ and found the value of J firstly with respect to the sections of $E \oplus E^\star$and secondly with respect to the sections of a Dirac structure D on E. Also I have considered the map $T : (\Gamma(E \oplus E^\star))^3 \rightarrow \mathbb{R}$, given by $T((s,\alpha),(v,\beta),(w,\gamma)) = \langle [(s,\alpha),(v,\beta)]_C,(w,\gamma)\rangle_+$. Next, I have determined several equivalent definitions for $T|_{\mathcal{D}}$ and I have also determined the relation between $J_2|_D$ and $T|_D$, and that $T|_D$ is a totally skew-symmetric tensor field. Based on these results I determined a first condition equivalent to integrability of Dirac structure, i.e. a Dirac structure D is integrable if and only if the map $T|_D$ vanishes on the sections of D. By demanding that J_2 to vanish on the sections of a Dirac structure, I obtained the main result of this section , i.e. a Dirac structure is integrable if and only if it can be endowed with a Lie algebroid structure. Among the first consequences of this result we have that Dirac structure D_ω is integrable if and only if the 2-form ω is closed and that the Dirac structures defined on TM and $T^\star M$, where M is a finite dimensional manifold, by means of non-linear connections are integrable if and only if those connections are flat. Then, by assuming that the Lie algebroid E is reflexive and that the skew-symmetric morphism $B : E^\star \rightarrow E$ has the property that the Dirac structure $D_B = graphB$ is integrable then $\Lambda^1(E)$ can be structured as Lie algebra.

We say that a function $f \in \mathcal{F}(E)$ is admissible with respect to a Dirac structure D, on E, if $d_E f \in \rho^\star(\Gamma(\mathcal{D}))$, or equivalently, if there is a section $X_f \in \rho(\Gamma(\mathcal{D}))$ such that $e_f = (X_f, d_E f) \in \Gamma(\mathcal{D})$. We denote the set of admissible functions on E, with respect to D by $\mathcal{F}_D(E)$. On $\mathcal{F}_D(E)$ I define the map $\{,\}_D$, given by $\{f,g\}_D = d_E f(X_g)$, where f, $g \in \mathcal{F}_D(E)$. Then, I showed that the map $\{,\}_D$ is well defined and that if D is integrable then the pair $(\mathcal{F}_D(E), \{,\}_D)$ is a Poisson algebra , on E. In this manner we extended to the case of a Lie algebroid the Proposition *2.5.1* from [8], which states that if M is a finite dimensional manifold and D is an integrable Dirac structure on M, then the pair $(\mathcal{F}_D(M), \{,\}_D)$ is a Poisson algebra. Here, the bracket $\{,\}_D$ is given by $\{f,g\}_D = X_g(f)$, where f, $g \in \mathcal{F}_D(M)$. Then I concluded that if D is an integrable Dirac structure on the Banach manifold M, then the pair $(\mathcal{F}_D(M), \{,\}_D)$ is a Poisson algebra . Furthermore, we showed that the sets of admissible functions with respect to the integrable Dirac structures D_ω, D_A and D_B are Poisson algebras where D_ω is given above and D_A and D_B are the graphs of the skew-symmetric morphisms $A : E \rightarrow E^\star$ and $B : E^\star \rightarrow E$.

In the first part of the third section, we have the definition of the Courant algebroid as given by *A. Weinstein*, *P. Xu* and *J.Z. Liu*, in [20], by a means of a five axiom definition. Later, in [37], *K. Uchino* showed that in order to define a Courant algebroid only three of the five axioms are needed. Here, together with *M. Anastasiei*, I have extended the definition of *S. Waldmann* and *F. Keller* to the case of anchored Banach bundles and have determined some of their basic properties and then we have shown that if E is a Lie algebroid then $E \oplus E^{\star}$ is a Courant algebroid. If we take $E = TM$, we see that $TM \oplus T^{\star}M$ is a Courant algebroid. In the second part of the third section we define the notion of Dirac structure with respect to a Courant algebroid and conclude that D is an integrable Dirac structure on a Lie algebroid E iff D is a Dirac structure on algebroidul Courant algebroid $E \oplus E^{\star}$.

The fourth chapter, entitled *Stokes-Dirac Structures* is divided into two sections, *Stokes-Dirac Structures* and *Hodge-Dirac Structures*. These structures are defined with respect to smooth finite dimensional manifolds with regular border. Let M be an m-dimensional smooth orientable manifold with regular boundary, denoted by ∂M. We consider spaces of the form $\mathcal{F}_{p,q} \times \mathcal{E}_{p,q}$, where $p + q = m + 1$and $\mathcal{F}_{p,q} = \Lambda_c^p(M) \times \Lambda_c^q(M) \times \Lambda_c^{m-p}(\partial M)$ and $\mathcal{E}_{p,q} = \Lambda_c^{m-p}(M) \times \Lambda_c^{m-q}(M) \times \Lambda_c^{m-q}(\partial M)$. Due to the principle of duality of Poincaré the vector bundle $\mathcal{F}_{p,q} \times \mathcal{E}_{p,q}$ is endowed a bilinear , symmetric and non-degenerate pairing $\langle , \rangle_+$. We denote by $D^{\perp}$ the ortho-complent of a vector sub-bundle D of $\mathcal{F}_{p,q} \times \mathcal{E}_{p,q}$, with respect to $\langle , \rangle_+$. Therefore we say that $D \subset \mathcal{F}_{p,q} \times \mathcal{E}_{p,q}$ is a Dirac structure on M if $D = D^{\perp}$.

The first remarkable examples of such structures are due to *A.J. van Schaft* and *B.M. Maschke*. With respect to this type of Dirac structures we define the notion of distributed port - Hamiltonian system and determine the conservation laws of such a system .

In the second part , using his Hodge $\star$ operator , I define Hodge - Dirac structures, and that of distributed port - Hamiltonian system corresponding to these structures and show that the energy of such a system is conserved along certain paths.

In the fifth chapter, called The Integrability of Stokes - Dirac Structures , I have firstly introduced the notions of almost Dirac structure and integrable almost Dirac structure by extending the definition of the integrable Dirac structure from the case of the Lie algebroid. Then we define the notion of integrable Stokes - Dirac structure.

Let $D \subset \mathcal{F}_{p,q} \times \mathcal{E}_{p,q}$ be a sub-bundle and consider the sub-bundles $F = \Lambda_c^p(M) \times \Lambda_c^{m-p}(M) \times \Lambda_c^{m-p}(\partial M)$ and $E = \Lambda_c^q(M) \times \Lambda_c^{m-q}(M) \times \Lambda_c^{m-q}(\partial M)$ as well as the canonical projections $\pi_p : \mathcal{F}_{p,q} \times \mathcal{E}_{p,q} \to F$ and $\pi_q : \mathcal{F}_{p,q} \times \mathcal{E}_{p,q} \to E$. The sub-bundles F and E are endowed with the symmetric bilinear maps and $\langle , \rangle_{p,+}$ and $\langle , \rangle_{q,+}$ and say that a sub-bundle D of $\mathcal{F}_{p,q} \times \mathcal{E}_{p,q}$ is an almost Dirac structure if $\pi_p(D)$ and $\pi_q(D)$ are Dirac structures . After that I have introduced the notion of integrable almost Dirac structure with respect to a pair of brackets $[,]_{p,0,C}$ and $[,]_{0,q,C}$. In particular, when M is a 3 - dimensional Riemannian manifold and p = q = 2 I explicitely defined the brackets $[,]_{2,0,C}$ and $[,]_{0,2,C}$, and anchors $\blacksquare_p$ and ρ_q. Afterwards, I have determined the expressions of the jacobiators of $[,]_{2,0,C}$ and $[,]_{0,2,C}$. Furthermore, I established several conditions equivalent with the integrability of the sub-bundles $\pi_p(D)$ and $\pi_q(D)$, respectively.

By using the pairings $[,]_{2,0,C}$ and $[,]_{0,2,C}$ I have defined the notion of integrable Dirac structure. Also, I have written the integrability conditions for Stokes - Dirac structures, and the Hodge Dirac structures, too.

Bibliografie

[1] Anastasiei, M., *Geometry of Lagrangians and semisprays on Lie algebroids*, Proceedings of the 5th Conference of Balkan Society of Geometers, 10–17, BSG Proc., 13, Geom. Balkan Press, Bucharest, 2006.

[2] Anastasiei, M., *Banach Lie algebroids,* An. Ştiinţ. Univ. Al. I. Cuza Iaşi. Mat. (N.S.), 57 (2), (2011), 409–416.

[3] Anastasiei, M., *Dirac structures in Banach Courant algebroids*, ROMAI J. 8 (2), (2012), 1–6.

[4] Anastasiei, M., Sandovici, A., Banach Dirac Bundles, IJGMMP 10 (7), (2013), 1350033 (16 pages) DOI: 10.1142/S0219887813500333.

[5] Anastasiei, M., Vulcu, V.-A., *On a Class of Courant Algebroids*, Submitted.

[6] Blankenstein, G., Ratiu, T., *Singular reductuion of implicit Hamiltonian systems*, Rep. Math. Phys. 53-2 (2004), 211-260.

[7] Boucetta, M., *Riemannian Geometry of Lie Algebroid*s, J. Egypt. Math. Soc., 19 (1-2), (2011), 57-70.

[8] Courant, T. J., *Dirac Manifolds*, Trans. Amer. Math. Soc., 319 (2), (1990), 631–661.

[9] Courant, T. J., Weinstein, A., Beyond Poisson structures, Action hamiltoniennes de groupes, Troisième théorème de Lie (Lyon, 1986), 39–49, Travaux en Cours, 27, Hermann, Paris, 1988.

[10] Crasmareanu, M., *Dirac Structures From Lie Integrability*, IJGMMP, 9 (4), (2012), 1220005, 7 pp., DOI:10.1142:S0219887812200058.

[11] Dorfman I., *Dirac structures and integrability of nonlinear evolution equations, Nonlinear Science: Theory and Applications,* John Wiley & Sons, Ltd., Chichester, 1993, xii+176 pp.

[12] Fernandes, R. L., *Lie algebroids, holonomy, and characteristic classes*, Adv. Math. 170 (1), (2002), 119–179.

[13] Guillemin, V., Sternberg, S, *Symplectic techniques in Physics*, Cambridge Univ. Press, New York, (1984).

[14] Gualtieri, M., *Generalized complex geometry*, Ph.D. thesis, Univ. Oxford, 2003, arXiv:math.DG/0401221.

[15] Hitchin, N. J., *Generalized Calabi-Yau manifolds*, Quart. J. Math., 54 (2003), 281-308.

[16] Kirillov, A. A., *Local Lie Algebras*, Russ. Math. Surv, 31(4), 55-75.

[17] Kosmann-Schwarzbach, Y., *Derived Brackets*, Lett. Math. Phys. 69 (2004), 61–87.

[18] Lang, S., *Fundamentals of Differential Geometry,* Springer-Verlag, New-York (1999).

[19] Lichnerowicz, A., *Le varietes de Poisson et leurs algebres de Lie asociees*, J. Diff. Geom., 12 (1977), 253-300.

[20] Liu, Z. J., Weinstein, A., Xu, P., *Manin triples for Lie bialgebroids*, J. Diff. Geom., 45 (1997), 547-574.

[21] Mackenzie, K., *Lie grupoids and Lie algebroids in differential geometry*, Lecture Notes Series, 134, London Math. Soc, Cambridge Univ. Press, (1987).

[22] Marsden, J. E., Ratiu, T. S., Abraham, R., *Manifolds, Tensor Analysis, and Applications*, 3rd Edition, 2007.

[23] Martinez, E., *Lagrangian mechanics on Lie algebroids*, Acta Appl. Math., 67 (2001), 295-320.

[24] Miron, R., *The geometry of High-Order Hamilton Spaces, Applications to Hamiltonian Mechanics*, Kluer Acad. Pub., 2003.

[25] Miron, R., Anastasiei, M., *Vector Bundles and Lagrange Spaces with Applications to Relativity*, Geom. Balkan Press, Bucharest, 1997.

[26] Miron, R., Hrimiuc, D., Shimada, H., Sabău, S., *The geometry of Hamilton and Lagrange Spaces*, Kluer Acad. Pub, New York, Boston, Dordrecht, London, Moscow, 2002.

[27] Miron, R., Anastasiei, M., Bucataru, I., *The Geometry of Lagrange Spaces. Handbook of Finsler Geometry*, vol. II, 969-1122, Kluwer Academic Publisher, 2003.

[28] Nishida, G., Yamakita, M., *A High order Stokes-Dirac structure for distributed parameter port-Hamiltonian systems*, Procs. of the 2004^{th} Amer. Control Conf., Boston, 2004.

[29] Gheorghiev, G., Oproiu, V., *Varietăţi diferenţiabile finit şi infinit dimensionale*, vol I-II, Ed. Acad., 1976.

[30] Pradines, J., *Theorie de Lie pour les groupoides differentiables. Calcul differentiel dans la categorie des groupoıdes infinitesimaux*, C. R. Acad. Sci. (Paris) 264 A (1967), 245-248.

[31] Maxim-Raileanu, L., *Cohomology of Lie Algebroids*, An. Sti. Univ. "Al. I. Cuza" Iasi, Sect. I-a Mat. (N.S.) 22 (2), (1976),179-199.

[32] Roytenberg, D., *Courant algebroids, derived brackets, and even symplectic supermanifolds*, Ph.D. Thesis, UC Berkley, 1999, math.DG/9910078.

[33] Schaft A. J., Maschke B. M., *Hamiltonian formulation of distrubuted- parameter systems with boudary energy flow*, J. Geom. Phys., 42 (2002), 166-194.

[34] Schaft, A. J., I*mplicit Hamiltonian systems with symmetry*, Rep. Math. Phys. 41 (1998), 203–221.

[35] Schaft, A. J., Blankenstein, G., Symmetry and Reductuion in Implicit Generalized Hamiltonian Systems, Memorandum no. 1489, 1999.

[36] Sniatycki, J., *Dirac brackets in geometric dynamics*, Ann. Inst. H. Poincare, Ann. Non Lineaire, 20 (1974), 365-372.

[37] Uchino, K., *Remarks on the definition of a Courant algebroid*, Lett. Math. Phys. 60 (2), (2002), 171–175.

[38] Vaisman, I., *Isotropic subbundles of* $TM \oplus T^{\star}M$, IJGMMP, 4 (3), (2007), 487-516.

[39] Vaisman, I., *Dirac structures on generalized Riemannian manifolds, Rev. Roumaine Math. Pures Appl., 57 (2), (2012), 179-203.*

[40] Vaisman, I., *Lectures on the geometry of Poisson manifolds*, Progress in Math. vol. 118, Birkhauser Verlag, Boston, 1994.

[41] Vulcu, V.-A., *On the integrability of a Stokes-Dirac structure*, ROMAIJ. 9 (1), (2013), 199–212.

[42] Vulcu, V.-A., Dirac Structures on Lie algebroids, Acceptată la Analele Universităţii "Ovidius" din Constanţa.

[43] Vulcu, V.-A., *Dirac Structures on Lie and Courant algebroids*, Comunicată la "Colloquium on Differential Geometry and Its Applications", Debrecen, 2013.

[44] Vulcu, V.-A., *Poisson Algebras from Dirac Structures on Banach Lie and Courant Algebroids*, 13^{th} International Conference of the Tensor Society on Differential Geometry and its Applications, and Informatics Besides, Iaşi, 2013.

[45] Waldmann, S., Keller, F., *Formal Deformations of Dirac Structures*, J. Geom. Phys., 3 (2007), 1015-1036.

[46] Weinstein, A., *The local structure of Poisson manifolds*, J. Diff. Geom., 18 (1983), 523-557.